a more exact formula like shown in Section 3.5.2, the actual power for $r = 2$ replicates is closer to 0.94 than 0.95. However, this approximate formula is accurate enough for most planning purposes.

The simple formula can also be used backwards by solving for Δ as a function of N, that is, $\Delta = 8 \times \sigma / \sqrt{N}$. That way, if an experimenter knows his budget for experimentation, which dictates the largest N can be, he can calculate the size of the effect Δ that he is likely to be able to detect. If the experimenter does not have an accurate estimate of σ, the formula can still be used by talking about practical effect size in units of the unknown σ. For example, if an experimenters' budget allows him to make at most $N = 64$ experiments, he can hope to detect effects that are no more than one standard deviation of the experimental error, that is, $\Delta = 8 \times \sigma / \sqrt{64} = \sigma$. This result will be true regardless of the number of factors in the two-level experiment. Consequently, with 64 runs he may have one factor with $r = 32$ replicates of each level, or six factors with $r = 1$ replicate of each of the $2^6 = 64$ treatment combinations.

3.7.5 Analysis with One Replicate per Cell

Factorial designs with one replicate per cell are often referred to as *unreplicated* designs. When there is adequate power for detecting effects with $r = 1$ replication per cell, or treatment combination, there is no need to double the experimental work by replicating each experiment. However, in an unreplicated factorial, the same problem arises that was discussed in Section 3.5.4. There will be zero degrees of freedom for calculating ssE and thus no F-tests for the effects. However, when there are multiple factors in a two-level factorial, there are simple graphical tools that allow detection of the significant effects. Since not all main effects and interactions in a 2^k experiment are expected to be significant, the levels of insignificant factors and combinations of levels defined by the insignificant interactions are equivalent to having replicates in the design. Graphical tools allow the significant effects (or equivalently regression coefficients) to be recognized.

The most common graphical tool used to spot significant effects are normal or half-normal plots that were first suggested by Daniel (1959). These are easy to produce using the `DanielPlot` function in the R package `FrF2` (Groemping, 2011a) or the `LGB` function in the package `daewr`. Additional graphical tools such as Lenth Plots and Bayes Plots are also useful for detecting significant effects and interactions and can be generated using functions in the `BsMD` package (Barrios, 2009). These graphical tools are also available for interactive analysis via the R DoE plugin (Groemping, 2011b) for the graphical user interface for R called R Commander (Fox, 2005). Examples of the menu selections and output available for analysis of two-level factorials using the R Commander DoE plugin are shown in files on the web page (`https://lawsonjsl7.netlify.app/webbook/`) for this book.

To illustrate the analysis of an unreplicated two-level factorial, consider an example from the chemical industry. Experimental design principles were